Sanjay Ghodawat University, Kolhapur

2020-21

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EXM/P/09/00

Year and Program: 2020-21 and SY M.Sc.	**School of Science**	**Department: Mathematics**
Course Code: MTS 601	**Course Title: Functional Analysis**	**Semester : III**
Day : **Date :**	**End Semester Examination**	**Time : 2 Hrs.** **Max Marks : 50**

Question Paper

Instructions: 1) All Questions are compulsory.

		Marks	Bloom's Level	CO
Q.1	**Answer the following (any Two)**–			
a)	Let M be a closed Linear Subspace of a normed linear space N. If the norm of coset x+M in the quotient space $\frac{N}{M}$ is defined by $\|x+M\| = inf\{\|x+M\|/m \in M\}$ then prove that $\frac{N}{M}$ is a normed Linear Space.	8	L3	CO1
b)	Prove that let N and N' be a normed linear space and Let $T \in B(N, N')$ (i.e. T is bounded Linear transformation N to N') Define $\|T\|$ by $\|T\| = \sup\{\|T(x)\|/x \in N \& \|x\| \leq 1\}$ the $\|T\|$ of T is also given by i) $\|T\| = \sup\{\|T(x)\|/x \in N \& \|x\| = 1\}$ ii) $\|T\| = \sup\left\{\frac{\|T(x)\|}{\|x\|}/x \in N \& x \neq 0\right\}$ iii) $\|T\| = \inf\{k/k \geq 0 \ \& \ \|T(x)\| \leq k\|x\|, \forall x \in N\}$.	8	L3	CO1
c)	Show that let N be a normed linear space a linear functional f on N is bounded (i.e. continuous) iff kerf $=\{x \in N/f(x) = 0\}$ is closed in N	8	L2	CO2
Q.2	**Answer the following (any two)** –			
a)	State and prove Hahn-Banach theorem	8	L3	CO2

b)	Let B be a Banach space and Let M and N be a closed linear subspace of B such that B = M $\oplus$ N ,if z = x+ y is the unique representation of vector in B as a sum of vectors in M and N ,then prove that mapping P defined by p(z)=x is a projection on B whose range and null space are M and N.	8	L4	CO3
c)	State and prove closed graph theorem.	8	L2	CO3

Q.3 Answer the following (any two) –

a)	Let H be a Hilbert space and let f be a arbitrary functional in H^* then prove that $\exists$ a unique vector y $\in H$ such that f(x) =(x, y) $\forall\, x \in H$ and $\|f\| = \|y\|$	9	L6	CO4
b)	State and prove Schwartz inequality.	5	L3	CO4
	Prove that let T be a normal operator on H then T(x)= λ x iff T^*(x)= $\bar{\lambda}$ x .	4	L2	CO5
c)	State and Prove spectral theorem	9	L5	CO5

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